Comparative Study of Histogram Equalization Algorithms for Image Enhancement

Li Lu*[a], Yicong Zhou[a], Karen Panetta[a], Sos Agaian[b]
[a] Department of Electrical and Computer Engineering, Tufts University, Medford, MA 02155
[b] Department of Electrical and Computer Engineering, University of Texas at San Antonio, San Antonio, TX 78249

ABSTRACT

Histogram equalization is one of the common tools for improving contrast in digital photography, remote sensing, medical imaging, and scientific visualization. It is a process for recovering lost contrast in an image by remapping the brightness values in such a way that equalizes or more evenly distributes its brightness values. However, Histogram Equalization may significantly change the brightness of the entire image and generate undesirable artifacts. Therefore, many Histogram Equalization based algorithms have been developed to overcome this problem. This paper presents a comprehensive review study of Histogram Equalization based algorithms. Computer simulations and analysis are provided to compare the enhancement performance of several Histogram Equalization based algorithms. A second-derivative-like enhancement measure is introduced to quantitatively evaluate their performance for image enhancement.

Keywords: Histogram equalization, image enhancement, spatial domain, transform domain, second-derivative-like measure of enhancement.

1. INTRODUCTION

Contrast enhancement is an important process in many imaging applications such as medical image analysis, remote sensing, advanced photography, and display technologies. Many different image processing techniques have been utilized for contrast enhancement [1-7]. Histogram equalization (HE) is a well-known technique for image enhancement due to its simplicity and effectiveness [1]. HE first calculates the occurrences of each intensity value in an image, then flattens and stretches the dynamic range of the histogram based on the probability density function [8]. It uses a monotonic, non-linear mapping to re-distribute the intensity values of pixels in the image. This mapping function is the cumulative distribution function (CDF) of the normalized image histogram [4]. The resulting output image contains a more uniform distribution of intensities. By this manner, HE increases the dynamic range of the histogram of the image while preserving the median value of the image. Thus the image contrast is highly improved [3]. However, images processed by the HE are seldom used in TV and video because they suffer from two disadvantages: over-enhanced high frequency gray level image region and loss of contrast in low frequency gray level image region [9]. Thus, many improved HE-based algorithms have been developed to overcome these two problems. They work either in the spatial domain or in the transform domain.

The spatial domain methods can be classified into two basic categories according to the transformation function: the global HE (GHE) [10-12] or the local HE (LHE) [5-7, 13-21]. GHE is a simple and fast method. It is mainly based on either image decomposition or histogram mapping. GHE which is based on image decomposition, segments the original image into several sub-images according to the pixel intensity levels, applies a HE process for all or part of sub-images, and then combines all sub-images to obtain the final enhanced image [22-32]. Histogram mapping based GHE maps the pixel intensity distribution of the original image into a reference histogram in order to change the image contrast [4, 33-42]. However, The GHE performance for contrast enhancement is relatively poor. On the other hand, the LHE uses a local histogram formed from localized data, rather than the global data, and a sliding window, which defines surrounding pixels, for contrast enhancement. In this way, the contrast of regions of interest can be improved in the enhanced image [5-7, 13-19]. Nevertheless, The computation complexity of the LHE is quite high due to its fully overlapped sub-blocks [1].

* Li.Lu@tufts.edu, phone 1-339-970-4214; fax 1-617-627-3220

Mobile Multimedia/Image Processing, Security, and Applications 2010, edited by Sos S. Agaian, Sabah A. Jassim,
Proc. of SPIE Vol. 7708, 770811 · © 2010 SPIE · CCC code: 0277-786X/10/$18 · doi: 10.1117/12.853502

Instead of considering pixel intensity distribution or the spatial relationship among the pixels and their surroundings, HE-based algorithms in the transform domain are intended to modify the distribution of different transform coefficients of the images using different image processing techniques. These transforms include the Discrete Cosine Transform (DCT)[43], Fast Fourier Transform (FFT), Discrete Wavelet Transform (DWT) [44] and also fuzzy set[45].

In this paper, a comprehensive review of HE-based enhancement approaches is addressed. We compare the enhancement performance of five HE-based algorithms, namely conventional HE [46], Bi-Histogram Equalization (BBHE) [26], Human Visual System Based Multi-Histogram Equalization (HVS_HE) [47], Logarithmic Transform Histogram Shifting (LTHS) [43] and Logarithmic Transform Histogram Matching (LTHM) [43]. A second-derivative-like measure of enhancement (SDME) is introduced to quantitatively evaluate their enhancement performance.

The rest of this paper is organized as followed. HE-based algorithms in the spatial and transform domains are addressed in Section 2 and Section 3, respectively. Section 4 introduces the new SDME measure. Section 5 compares five HE-based algorithms for image enhancement and conclusion is drawn in Section 6.

2. Histogram Equalization in the Spatial Domain

Spatial domain based HEs improve contrast by changing the pixel intensity distribution or intensity levels. They are generally classified into two categories: GHE and LHE. GHE-based algorithms enhance images based on different techniques such as Conventional HE, Histogram Mapping, which transform the image pixel intensity distribution into a desired reference histogram, and image decomposition, which decomposes the original image into several sub-images based on intensity levels. LHE usually processes an image locally by changing the pixel intensity values based on the properties of the local sliding window that the pixel belongs to.

2.1. Conventional HE

Conventional HE tends to flatten and stretch the dynamic range of image's histogram to produce an image with better contrast [48]. Suppose the image x has the number of occurrences of gray level *i*. The probability of an occurrence of a pixel of level *i* in the image is

$$p_x(i) = \frac{n_i}{n}, \quad 0 \le i < L \tag{1}$$

L is the maximum of gray levels in the image, *n* is the total number of pixels in the image, and $p_x(i)$ is in fact the image's histogram for pixel value *i*, normalized to [0,1].
The corresponding CDF is

$$CDF(i)_x = \sum_{j=0}^{i} p_x(j) \tag{2}$$

The CDF then will be linearized across the value range like

$$CDF(i)_y = iK \tag{3}$$

For some constant K, the properties of the CDF allow to perform such a transform

$$y = CDF(x)_x \tag{4}$$

where y is in the range [0, 1]. In order to map the values back into their original range, the following simple transformation needs to be applied on the result:

$$y' = y \cdot (\max\{x\} - \min\{x\}) + \min\{x\} \tag{5}$$

2.2. Decomposition-based HEs

Decomposition-based HEs usually segment the input image into several sub-images with respect to the pixel intensity levels. BBHE, proposed by Kim [9], is a method preserving the mean value of the image by dividing the image histogram into two sub-histograms based on the mean value and equalizing them independently. Dualistic sub-image histogram equalization (DSIHE) proposed by Wang et al [49], partitions the image into two sub-images based on the median value of the image. Minimum mean brightness error bi-histogram equalization (MMBEBHE) [3], divides the input image into two sub-images with the smallest Absolute Mean Brightness Error (AMBE). Sometimes two sub-images could not meet the requirement of maintaining the brightness without over-enhancement. Thus, the decomposition-based HEs segment images into more than two sub-images. Examples include Multi-peak histogram equalization (MPHE) [50], recursive sub-image histogram equalization (RSIHE) [36], Recursive Mean Separate Histogram Equalization (RMSHE) [51], Dynamic Histogram Equalization (DHE) [52], brightness preserving dynamic histogram equalization (BPDHE) [53], Brightness Preserving Weight Clustering Histogram Equalization (BPWCHE) [30] Human Visual System based Histogram Equalization (HVS_HE) [31], Contrast Stretching Recursive Separated Histogram Equalization (CSRSHE) [29], and sub-regions histogram equalization. DHE segments image histogram based on local minima and assigns specific gray level range for each sub-image. It controls over the effect of conventional HE so that it can enhance the image without incurring any loss of details in the image. CSRSHE changes pixel values according to global and local statistics of the image and then performs recursive mean separate histogram equalization using a modified local contrast stretching manipulation. As a result, brightness can be preserved more accurately and better contrast can be achieved. SRHE segments image based on the smoothed intensity value obtained from the convolution of the input image and a Gaussian filter. It can not only enhance the image contrast but also sharpen the edges in the image.

As an example of decomposition-based HEs, HVS_HE segments the original image into four sub-images based on based on four defined regions with different background intensity: (1) the saturation region for over-illuminated areas; (2) the Weber region for properly illuminated areas; (3) the Devries–Rose region for under-illuminated areas; (4) the fourth region for all pixels underneath the curve containing the least informative pixels [54]. The four regions are shown in Fig. 1. Each of these regions can be enhanced separately and recombined to form a more visually pleasing enhanced image.

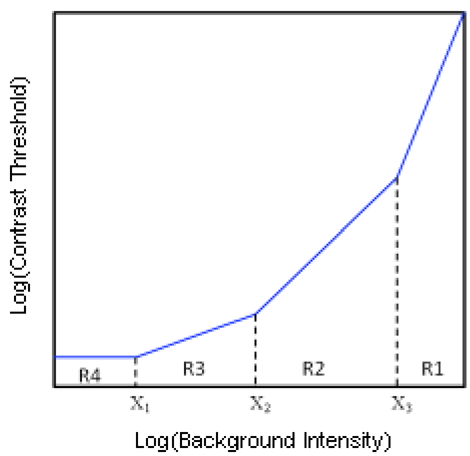


Fig.1: Four VHS-based regions. Devries–Rose from x1 to x2, Weber from x2 to x3, saturation from x3 to infinity, and the fourth region below the line, containing the least informative pixels [55].

The background intensity of the Human Visual System is defined as a weighted local mean [56],

$$B(x,y) = l[l(\frac{l}{2}\sum_{Q} X(i,j) + \frac{q}{2}\sum_{Q'} X(i,j)) + X(x,y)] \tag{6}$$

where *B(x, y)* is the background intensity at each pixel, *X(x, y)* is the input image, *Q* is all of the pixels which are directly up, down, left, and right from the pixel, *Q'* is all of the pixels diagonally one pixel away, and *l* and *q* are some constant.

The segmentation theory is based on the formula below,

$$B_T = \max(X(x,y)) - \min(x,y)) \tag{7}$$

$$B_{X1} = \alpha_1 B_T \quad B_{X2} = \alpha_2 B_T \quad B_{X3} = \alpha_3 B_T \tag{8}$$

$$K_1 = \frac{1}{100}\beta \cdot \max(\frac{X'(x,y)}{B(x,y)}) \tag{9}$$

$$K_2 = K_1 \sqrt{B_{X2}} \tag{10}$$

$$K_3 = K_1 / B_{X3} \tag{11}$$

B_T, which is the maximum difference in the image, α_1, α_2, α_3 are parameters based upon the four different regions of response characteristics displayed by the human eye, X'(x, y) is the gradient of matrix. B(x, y) is the background intensity of the image. The four segmented images should satisfy requirements as below,

Im1=X(x,y), for

$$B(x,y) >= B_{X3} \;\&\; \frac{X'(x,y)}{B(x,y)} >= K_3 \tag{12}$$

Im2=X(x,y), for

$$B_{X2} >= B(x,y) >= B_{X1} \;\&\; \frac{X'(x,y)}{\sqrt{B(x,y)}} >= K_2 \tag{13}$$

Im3=X(x,y), for

$$B_{X3} >= B(x,y) >= B_{X2} \;\&\; \frac{X'(x,y)}{B(x,y)} >= K_1 \tag{14}$$

Im4=X(x,y) for

All remaining pixels;

HE is applied to each sub-image and then all the enhanced sub-images are combined to obtain the final enhanced image. The HVS_HE is able to address local features of an image using less complex global algorithms, relying on models of the HVS to ensure visually appealing output.

2.3. Histogram Mapping

Histogram mapping algorithms map the histogram of the input image to the target histogram, which is obtained by the histogram of the same image or from the intended histogram distribution of other resources. An alternative histogram mapping is called histogram shifting, which shifts the histogram several bins in specific direction rather than mapping the histogram to the target histogram directly. The method introduced in [41] fits a given histogram with any dimensions to the target distribution and therefore their squared error is derived and minimized. This multidimensional extension is quite natural because the method is formulated as a nonlinear optimization with bound constraints. In the Modified Segmental Histogram Equalization (MSHEQ) proposed in [42], the CDF is shifted to a higher value for transformation.

Generally, histogram mapping alters the image data such that the resulting histogram matches a specific desired curve [43]. This is also known as the histogram mapping and histogram specification [24]. The key point of histogram mapping lies at solving an equation that compares the integrals of the probability density function, basically comparing their CDF [43].

These three types of HE-based enhancement methods can be combined with each other to generate enhancement algorithms to produce better enhancement results. There are many enhancement methods that use more than one type of spatial domain histogram equalization techniques [40, 57-63]. Method Clipped Histogram Equalization (CHE) introduced in [40] first uses decomposition-based HE to segment the histogram with respect to a pre-specified upper limit, and then adopts histogram shifting to shift the entire processed histogram over the limit.

2.4. Region-based HEs

Region-based HEs, which belongs to the LHE, processes images based on the local properties of images using a sliding window. When the contrast varies in one image such as an image partly in shadow, the enhancement of GHEs will be highly limited. LHEs are then developed to cope with this problem. LHEs enhance images based on the local histogram of the images, generally within a sliding window around each pixel [7]. Many HE-based algorithms adopt this concept.

Adaptive Histogram Equalization (AHE) computes several local histograms with respect to neighborhoods around each pixel in the sliding windows, and uses them to redistribute the pixel intensity values. Contrast-limited adaptive Histogram Equalization (CLAHE), developed by Pizer [16], clips the histogram to a predefined histogram, and then redistributes the excess counts among the other intensities before computing the CDF. It improves AHE by limiting the amount of contrast within a local area with an adjustable parameter. The CLAHE is effective in enhancing contrast but creates an annoying image. Variable Region Adaptive Histogram Equalization (VRAHE) proposed by Vossepoel et al [17] is very similar to AHE except that it replaces each regional histogram that covers only a small range of grey-values by a linear combination of the histograms of neighboring regions, instead of averaging it with a uniform distribution. Moving Histogram Equalization (MHE) introduced in [6] is much faster than the AHE since less computation is required for the interpolation. Thresholding HE [33] was proposed to eliminate noisy pixels with gray levels less than the threshold value. It divides the histogram of the image into zones according to the gray level, and then maps each zone into a display intensity range using a specific transformation function. Blurred Weighted Adaptive Histogram Equalization (BWAHE) in [28] can be used to compute a two parametric families of images at varying contrast levels. Partially Overlapped Sub-block Histogram Equalization (POSHE) [17] introduced by Kim et al, uses a low-pass filter-type mask to get non-overlapped sub-block HE to produce high contrast associated with local HE but with simplicity of global HE. With this method the computation complexity can be reduced considerably and blocking effects eliminated.

BBHE [9, 26] in the group of region based HEs has been implemented for comparison. Suppose the mean value of the image X is X_m, then the image is decomposed into two sub-images based on the equations,

$$\begin{cases} X_L(i,j) < X_M & X(i,j) \in X \\ X_L(i,j) \geq X_M & X(i,j) \in X \end{cases} \tag{15}$$

Then HE is applied to each sub-image and then these two sub-images are combined to generate the final result.

3. Histogram Equalization in the Transform Domain

In the case of transform domain enhancement techniques [64], the image intensity data are mapped into a given transform domain by using a specific transform such as the 2-D discrete cosine transform (DCT) [43], Fourier Transform, wavelet-based [44], and fuzzy set [45]. Fundamentally, these algorithms enhance images by manipulating transform coefficients.

3.1. Logarithmic Transform Histogram Shifting (LTHS)

This method shifts in the logarithmic transform coefficient histogram of an image in the positive direction. By mapping the image into the shifted histogram and returning the data back into the spatial domain, the dynamic range of the image is expanded, improving contrast and enhancing details throughout [43]. The process of Logarithmic Transform Domain Coefficient Histogram Shifting introduced in [43] can be depicted in Fig. 2.

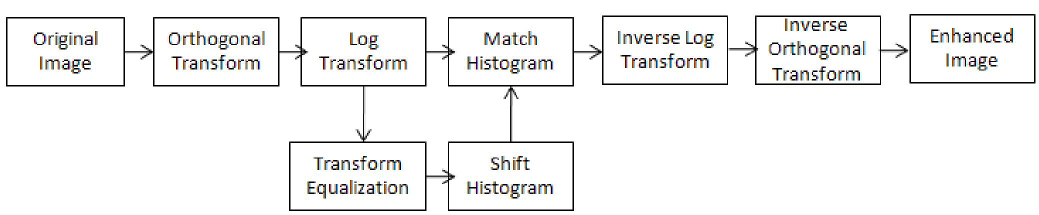


Fig. 2: Block diagram of logarithmic transform histogram shifting

3.2. Logarithmic Transform Histogram Matching (LTHM)

The process of Logarithmic Transform Domain Coefficient Histogram Matching introduced in [43] can be depicted as below:

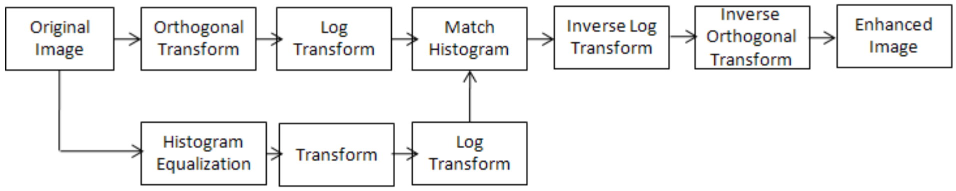


Fig. 3: Block diagram of logarithmic transform histogram matching with spatial equalization

First, HE has been applied to the image and then the logarithmic transform histogram is calculated. The original image also have its logarithm transform coefficients mapped to create a similar histogram to match the equalized image's transform histogram coefficients [43] as shown in Fig. 3.

4. Enhancement Measure

Since image enhancement algorithms are intended to improve image contrast, a contrast measure to assess the enhancement performance is introduced. To quantitatively evaluate the performance of HE-based algorithms for image enhancement, a second-derivative-like measure of enhancement (SDME) is introduced using the concept of the second derivative. The SDME is defined by,

$$SDME_{k1k2} = -\frac{1}{k_1 k_2}\sum_{l=1}^{k1}\sum_{k=1}^{k2} 20\ln\left|\frac{I_{\max;k,l} - 2I_{center;k,l} + I_{\min;k,l}}{I_{\max;k,l} + 2I_{center;k,l} + I_{\min;k,l}}\right| \tag{16}$$

where an image is divided into k1*k2 blocks. $I_{\max;k,l}$, $I_{\min;k,l}$ are the maximum and minimum values of the pixels in each block separately, and $I_{center;k,l}$ is the intensity of the center pixel in each block. Thus, the size of blocks should be odd number such as 3×3. Each block contains an odd number of pixels.

5. Simulation Results and Comparisons

In order to demonstrate the advantages and disadvantages of HEs introduced in Section 2 and Section 3, we compared five different HE-based algorithms, namely Conventional HE, BBHE in Region based HEs[9], HVS_HE [31] in Decomposition based HEs, LTHS and LTHM in the transform domain HEs. In the computer simulation, we have run eleven images which include a street view, natural picture, indoor view and medical image. Images named "Man", "Castle", "Bed", "Outside" are natural gray images and "Foot", "Breast", "Brain", "Prostate Cancer 1" and "Prostate Cancer 2" are medical images and their dimensions are equal or bigger than 256*256 pixels and smaller than 600*600 pixels. Seven images among these eleven images are selected as examples given in Fig. 4. Five HE based algorithms explained previously are implemented and compared. The new SDME measure is evaluate all the original and enhanced images. The procedures of this comparison listed as followed:

Step 1 Enhance input image with five HE based enhancement algorithms, separately;

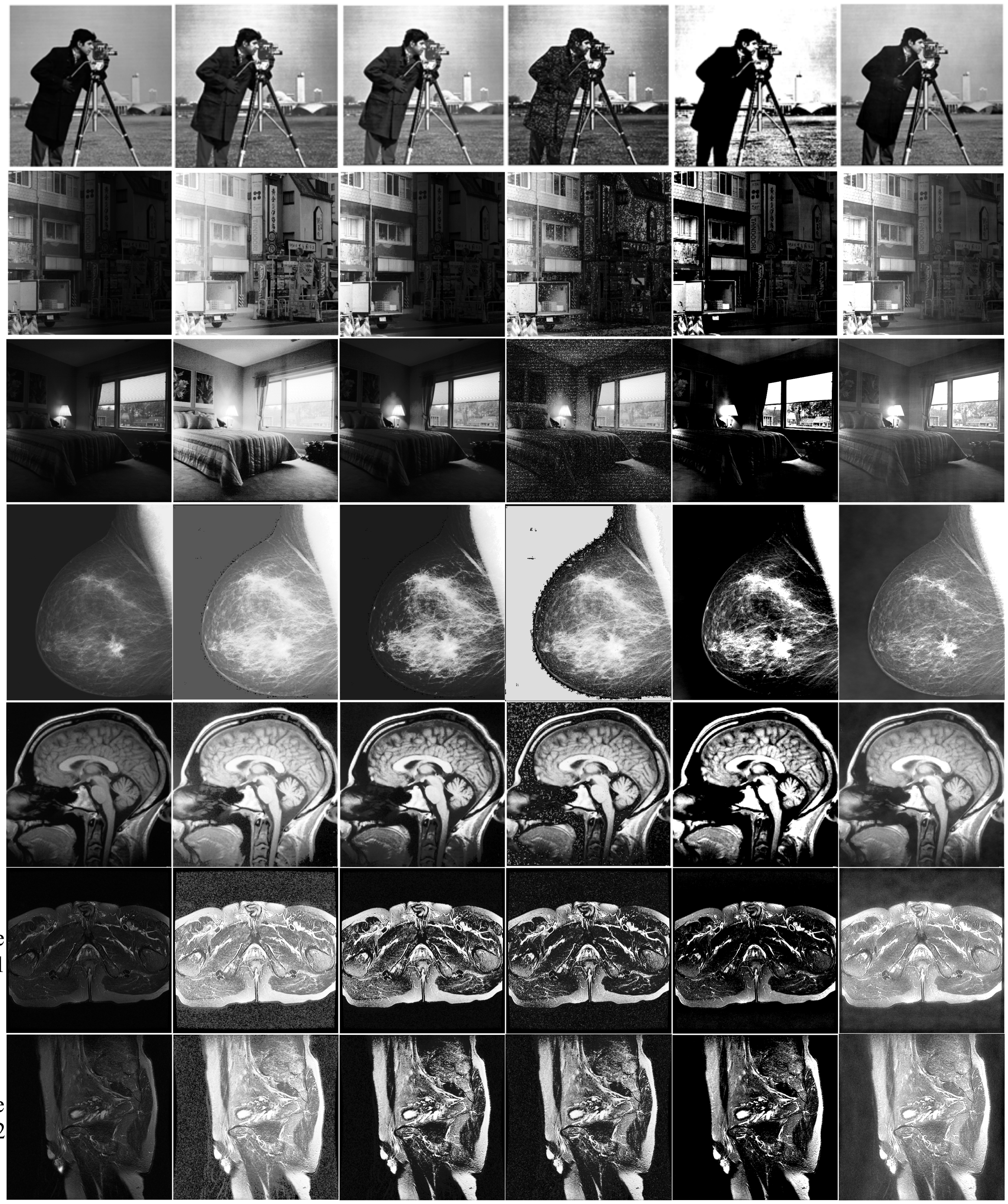
Step 2 Run SDME algorithm on the five enhanced images and original images, get the SDME values;

Step 3 Record six SDME values of each image, find the mean value of SDME of each algorithm;

Step 4 Compare SDME value with the visual qualities of enhanced images;

Step 5 Compare six SDME mean values;

Output Comparison conclusion drawn.



Man
Street
Bed
Breast
Brain
Prostate Cancer1
Prostate Cancer2

(a) (b) (c) (d) (e) (f)

Fig. 4: Image Enhancement comparison (a) Original Image (b) Conventional HE (c) BBHE (d)HVS_HE (e)LTHS (f) LTHM

From the enhanced results in Fig. 4, it can be seen that the images enhanced by the conventional HE suffer from disadvantages of over-enhancement and loss of contrast in low frequency regions. This can be found in the left side of "Street" image, the scene outside the window in image "Bed", and the middle part of image "Brain". BBHE, which can preserve the mean value of the entire image, can overcome over-enhancement problems when compared with the conventional HE. However, it suffers from a loss of contrast in the low frequency regions due to preserving the mean value of the image. In the results enhanced by the BBHE, there are more dark regions than those of the conventional HE.

HVS_HE can preserve edge details while enhancing contrast in images with non-uniform illumination, but it also introduces additional noise during the processes of segmentations and enhancement. The LTHS strengthens the contrast of low frequency gray level regions and high frequency level regions, but fails to reveal the details in low frequency gray level regions. The LTHM shows the best enhancement performance compared to other four enhancement algorithms. It enhances the image without over-enhancement and reveals many details in low contrast gray level regions.

The SDME measure results plotted in Fig. 5 are consistent with the enhanced results in Fig. 4. Images enhanced by the LTHM have the best SDME values compared to other four enhancement algorithms. Images enhanced with BBHE have higher SDME values when compared with images enhanced by conventional HE because they have less over-enhancement problems.

Table 1: SDME measure results of the original and enhanced images in Fig. 4

SDME	Original	HE	BBHE	HVS_HE	HTHS	LTHM
Man	33.6806	28.8798	33.5080	25.7205	23.1963	34.6875
Castle	29.7226	27.9296	29.8856	26.5402	16.9756	35.4388
Street	26.0449	27.2841	27.5358	19.9137	15.7475	34.3959
Bed	29.6214	29.6461	32.0506	19.6761	19.3052	41.3145
Outside	35.4671	29.8915	35.5016	27.1577	18.5843	39.0454
Foot	33.0946	33.3933	36.9607	26.0101	15.8349	43.8940
Breast	37.3556	36.7867	38.1638	32.5457	18.5017	47.3307
Brain	29.3328	28.9354	29.2713	23.2681	19.5086	35.4404
hand	31.5140	28.3446	30.4427	21.4844	16.8675	36.5478
Prostate Cancer1	26.2913	28.2665	23.9229	19.4403	15.1900	32.1792
Prostate Cancer2	28.5391	28.5937	26.2081	23.3540	20.2158	32.9405
Average value	30.9695	29.8138	31.2228	24.1010	18.1752	37.5650

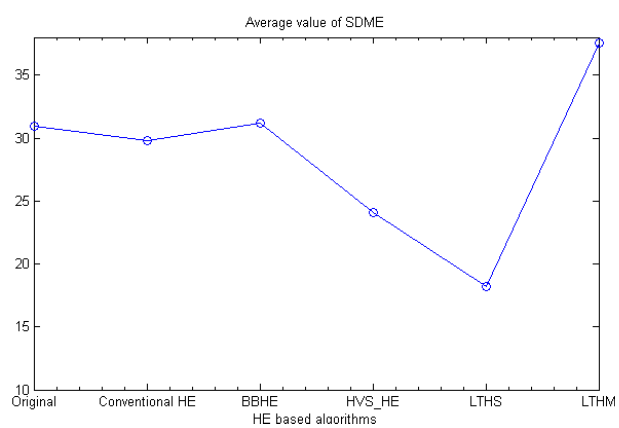


Fig. 5: The average of SDME measure results for each algorithm.

6. Conclusion

In this paper we have performed a comprehensive review study of histogram equalization algorithms for image enhancement. We have compared five different Histogram Equalization based algorithms. They include Conventional Histogram equalization, Bi-histogram equalization [26], Human Visual System Based Multi-Histogram Equalization

[47], Logarithmic Transform Histogram Shifting, and Logarithmic Transform Histogram Matching [43]. To quantitatively evaluate their enhancement performance, the second-derivative-like measure of enhancement has been also introduced. The experiment results show that the Logarithmic Transform Histogram Matching shows better overall enhancement performance compared to other four algorithms. It shows less over-enhanced high frequency components and better contrast in the lower frequency regions compared with other implemented algorithms.

ACKNOWLEDGEMENT

This work has been partially supported by NSF Grant HRD-0932339.

REFERENCES

[1] Kim, J-Y, Kim, L-S, and Hwang, S-H, "An advanced contrast enhancement using partially overlapped sub-block histogram equalization," *Circuits and Systems for Video Technology, IEEE Transactions on,* vol. 11, 475-484 (2001).

[2] Wang, C and Ye, Z, "Brightness preserving histogram equalization with maximum entropy: a variational perspective," *Consumer Electronics, IEEE Transactions on,* vol. 51, 1326-1334 (2005).

[3] Chen, S-D and Ramli, A R, "Minimum mean brightness error bi-histogram equalization in contrast enhancement," *Consumer Electronics, IEEE Transactions on,* vol. 49, 1310-1319 (2003).

[4] Jafar, I and Hao, Y, "Image contrast enhancement by constrained variational histogram equalization," in *Electro/Information Technology, 2007 IEEE International Conference on*, 120-125 (2007).

[5] Tian, Y, Wan, Q, and Wu, F, "Local Histogram Equalization Based on the Minimum Brightness Error," in *Image and Graphics, 2007. ICIG 2007. Fourth International Conference on*, 58-61 (2007).

[6] Leszczynski, K W and Shalev, S, "A robust algorithm for contrast enhancement by local histogram modification," *Image and Vision Computing,* vol. 7, 205-209 (1989).

[7] Stark, J A and Fitzgerald, W J, "An Alternative Algorithm for Adaptive Histogram Equalization," *Graphical Models and Image Processing,* vol. 58, 180-185 (1996).

[8] Chao, W and Zhongfu, Y, "Brightness preserving histogram equalization with maximum entropy: a variational perspective," *Consumer Electronics, IEEE Transactions on,* vol. 51, 1326-1334 (2005).

[9] Kim, Y-T, "Contrast enhancement using brightness preserving bi-histogram equalization," *Consumer Electronics, IEEE Transactions on,* vol. 43, 1-8 (1997).

[10] Avanaki, A, "Exact global histogram specification optimized for structural similarity," *Optical Review,* vol. 16, 613-621 (2009).

[11] Stark, J A and Fitzgerald, W J, "Model-based adaptive histogram equalization," *Signal Processing,* vol. 39, 193-200 (1994).

[12] Singhai, J and Rawat, P, "Image enhancement method for underwater, ground and satellite images using brightness preserving histogram equalization with maximum entropy," in *Conference on Computational Intelligence and Multimedia Applications, 2007. International Conference on*, 507-512 (2007).

[13] Pisano, E, Zong, S, Hemminger, B, DeLuca, M, Johnston, R, Muller, K, Braeuning, M, and Pizer, S, "Contrast Limited Adaptive Histogram Equalization image processing to improve the detection of simulated spiculations in dense mammograms," *Journal of Digital Imaging,* vol. 11, 193-200 (1998).

[14] Verdenet, J, Cardot, J C, Baud, M, Chervet, H, Duvernoy, J, and Bidet, R, "Scintigraphic image contrast-enhancement techniques: Global and local area histogram equalization," *European Journal of Nuclear Medicine and Molecular Imaging,* vol. 6, 261-264 (1981).

[15] Zimmerman, J, Cousins, S, Hartzell, K, Frisse, M, and Kahn, M, "A psychophysical comparison of two methods for adaptive histogram equalization," *Journal of Digital Imaging,* vol. 2, 82-91 (1989).

[16] Pizer, S M, Amburn, E P, Austin, J D, Cromartie, R, Geselowitz, A, Greer, T, ter Haar Romeny, B, Zimmerman, J B, and Zuiderveld, K, "Adaptive histogram equalization and its variations," *Computer Vision, Graphics, and Image Processing,* vol. 39, 355-368 (1987).

[17] Vossepoel, A M, Stoel, B C, and Meershoek, A P, "Adaptive histogram equalization using variable regions," in *Pattern Recognition, 1988., 9th International Conference on*, vol.1,351-353 (1988).

[18] Sherrier, R H and Johnson, G A, "Regionally Adaptive Histogram Equalization of the Chest," *Medical Imaging, IEEE Transactions on,* vol. 6, 1-7 (1987).
[19] Reza, A M, "Realization of the Contrast Limited Adaptive Histogram Equalization (CLAHE) for Real-Time Image Enhancement," *The Journal of VLSI Signal Processing,* vol. 38, 35-44 (2004).
[20] Jen, T-C and Wang, S-J, "Generalized Histogram Equalization Based on Local Characteristics," in *Image Processing, 2006 IEEE International Conference on*, 2877-2880 (2006).
[21] Paranjape, R B, Morrow, W M, and Rangayyan, R M, "Adaptive-neighborhood histogram equalization for image enhancement," *CVGIP: Graphical Models and Image Processing,* vol. 54, 259-267 (1992).
[22] Ibrahim, H and Kong, N S P, "Image sharpening using sub-regions histogram equalization," *Consumer Electronics, IEEE Transactions on,* vol. 55, 891-895 (2009).
[23] Chen, S-D and Rahman Ramli, A, "Preserving brightness in histogram equalization based contrast enhancement techniques," *Digital Signal Processing,* vol. 14, 413-428 (2004).
[24] Chitwong, S, Boonmee, T, and Cheevasuvit, F, "Local area histogram equalization based multispectral image enhancement from clustering using competitive Hopfield neural network," in *Electrical and Computer Engineering, 2003. IEEE CCECE 2003. Canadian Conference on*, vol.3, 1715-1718 (2003).
[25] Santos, V T, Schiabel, H, Góes, C E, and Benatti, R H, "A Segmentation Technique to Detect Masses in Dense Breast Digitized Mammograms," *Journal of Digital Imaging,* vol. 15, 210-213 (2002).
[26] Kim, Y-T, "Quantized bi-histogram equalization," in *Acoustics, Speech, and Signal Processing, 1997. ICASSP-97., 1997 IEEE International Conference on*, vol.4, 2797-2800 (1997).
[27] Dale-Jones, R and Tjahjadi, T, "A study and modification of the local histogram equalization algorithm," *Pattern Recognition,* vol. 26, 1373-1381 (1993).
[28] Gauch, J M, "Investigations of image contrast space defined by variations on histogram equalization," *CVGIP: Graphical Models and Image Processing,* vol. 54, 269-280 (1992).
[29] Jagatheeswari, P, Kumar, S S, and Rajaram, M, "Contrast Stretching Recursively Separated Histogram Equalization for Brightness Preservation and Contrast Enhancement," in *Advances in Computing, Control, & Telecommunication Technologies, 2009. ACT '09. International Conference on*, 111-115 (2009).
[30] Sengee, N and Heung, C, "Brightness preserving weight clustering histogram equalization," *Consumer Electronics, IEEE Transactions on,* vol. 54, 1329-1337 (2008).
[31] Wharton, E, Panetta, K, and Agaian, S, "Human Visual System Based Multi-Histogram Equalization for Non-Uniform Illumination and Shoadow Correction," in *Acoustics, Speech and Signal Processing, 2007. ICASSP 2007. IEEE International Conference on*, I-729-I-732 (2007).
[32] Delon, J, "Midway Image Equalization," *Journal of Mathematical Imaging and Vision,* vol. 21, 119-134 (2004).
[33] Chuang, K-S, Chen, S, and Hwang, I-M, "Thresholding Histogram Equalization," *Journal of Digital Imaging,* vol. 14, 182-185 (2001).
[34] Aviram, G and Rotman, S R, "Analyzing the improving effect of modeled histogram enhancement on human target detection performance of infrared images," *Infrared Physics & Technology,* vol. 41, 163-168 (2000).
[35] Wang, B-j, Liu, S-q, Li, Q, and Zhou, H-x, "A real-time contrast enhancement algorithm for infrared images based on plateau histogram," *Infrared Physics & Technology,* vol. 48, 77-82 (2006).
[36] Sim, K S, Tso, C P, and Tan, Y Y, "Recursive sub-image histogram equalization applied to gray scale images," *Pattern Recognition Letters,* vol. 28, 1209-1221 (2007).
[37] Ziaei, A, Yeganeh, H, Faez, K, and Sargolzaei, S, "A Novel Approach for Contrast Enhancement in Biomedical Images Based on Histogram Equalization," in *BioMedical Engineering and Informatics, 2008. BMEI 2008. International Conference on*, 855-858 (2008).
[38] O'Gorman, L, "A note on histogram equalization for optimal intensity range utilization," *Computer Vision, Graphics, and Image Processing,* vol. 41, 229-232 (1988).
[39] Kong, N S P, Ibrahim, H, Ooi, C H, and Chieh, D C J, "Enhancement of Microscopic Images Using Modified Self-Adaptive Plateau Histogram Equalization," in *Computer Technology and Development, 2009. ICCTD '09. International Conference on*, 308-310 (2009).
[40] Kim, T and Paik, J, "Adaptive contrast enhancement using gain-controllable clipped histogram equalization," *Consumer Electronics, IEEE Transactions on,* vol. 54, 1803-1810 (2008).
[41] Taemin, K and Yang, H S, "A Multidimensional Histogram Equalization by Fitting an Isotropic Gaussian Mixture to a Uniform Distribution," in *Image Processing, 2006 IEEE International Conference on*, 2865-2868 (2006).

[42] Skosan, M and Mashao, D, "Modified Segmental Histogram Equalization for robust speaker verification," *Pattern Recognition Letters,* vol. 27, 479-486 (2006).

[43] Agaian, S S, Silver, B, and Panetta, K A, "Transform Coefficient Histogram-Based Image Enhancement Algorithms Using Contrast Entropy," *Image Processing, IEEE Transactions on,* vol. 16, 741-758 (2007).

[44] Fu, J C, Lien, H C, and Wong, S T C, "Wavelet-based histogram equalization enhancement of gastric sonogram images," *Computerized Medical Imaging and Graphics,* vol. 24, 59-68 (2000).

[45] Vlachos, I K and Sergiadis, G D, "Hesitancy Histogram Equalization," in *Fuzzy Systems Conference, 2007. FUZZ-IEEE 2007. IEEE International*, 1-6 (2007).

[46] Hummel, R A, "Histogram modification techniques," *Computer Graphics and Image Processing,* vol. 4, 209-224 (1975).

[47] Panetta, K A, Wharton, E J, and Agaian, S S, "Human Visual System-Based Image Enhancement and Logarithmic Contrast Measure," *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on,* vol. 38, 174-188 (2008).

[48] Chen Soong Der and Sidhu, M S, "Re-evaluation of Automatic Global Histogram Equalization-based Contrast," *eJCSIT Electronic Journal of Computer Science & Information Technology,* vol. 1, 13-17 (2009).

[49] Wang, Y, Chen, Q, and Zhang, B, "Image enhancement based on equal area dualistic sub-image histogram equalization method," *Consumer Electronics, IEEE Transactions on,* vol. 45, 68-75 (1999).

[50] Wongsritong, K, Kittayaruasiriwat, K, Cheevasuvit, F, Dejhan, K, and Somboonkaew, A, "Contrast enhancement using multipeak histogram equalization with brightness preserving," in *Circuits and Systems, 1998. IEEE APCCAS 1998. The 1998 IEEE Asia-Pacific Conference on*, 455-458 (1998).

[51] Kim, M and Min, C, "Recursively separated and weighted histogram equalization for brightness preservation and contrast enhancement," *Consumer Electronics, IEEE Transactions on,* vol. 54, 1389-1397 (2008).

[52] Abdullah-Al-Wadud, M, Kabir, M H, Dewan, M A A, and Oksam, C, "A Dynamic Histogram Equalization for Image Contrast Enhancement," *Consumer Electronics, IEEE Transactions on,* vol. 53, 593-600 (2007).

[53] Ibrahim, H and Kong, N S P, "Brightness Preserving Dynamic Histogram Equalization for Image Contrast Enhancement," *Consumer Electronics, IEEE Transactions on,* vol. 53, 1752-1758 (2007).

[54] Kundu, M K and Pal, S K, "Thresholding for edge detection using human psychovisual phenomena," *Pattern Recognition Letters,* vol. 4, 433-441 (1986).

[55] Panetta, K A, Wharton, E J, and Agaian, S S, "Human Visual System-Based Image Enhancement and Logarithmic Contrast Measure," *Systems, Man, and Cybernetics, Part B, IEEE Transactions on,* vol. 38, 174-188 (2008).

[56] Wharton, E, Agaian, S, and Panetta, K, "A logarithmic measure of image enhancement," in *Mobile Multimedia/Image Processing for Military and Security Applications*, Orlando, FL, USA, 62500(2006).

[57] Chen Hee, O, Kong, N S P, and Ibrahim, H, "Bi-histogram equalization with a plateau limit for digital image enhancement," *Consumer Electronics, IEEE Transactions on,* vol. 55, 2072-2080 (2009).

[58] Qian, W, Liya, C, and Dinggang, S, "Fast histogram equalization for medical image enhancement," in *Engineering in Medicine and Biology Society, 2008. EMBS 2008. 30th Annual International Conference of the IEEE*, 2217-2220 (2008).

[59] Cheng, H D and Shi, X J, "A simple and effective histogram equalization approach to image enhancement," *Digital Signal Processing,* vol. 14, 158-170 (2004).

[60] Abdullah-Al-Wadud, M, Kabir, M H, and Chae, O, "A spatially controlled histogram equalization for image enhancement," in *Computer and Information Sciences, 2008. ISCIS '08. 23rd International Symposium on*, 1-6 (2008).

[61] Stefanoyiannis, A P, Costaridou, L, Skiadopoulos, S, and Panayiotakis, G, "A digital equalisation technique improving visualisation of dense mammary gland and breast periphery in mammography," *European Journal of Radiology,* vol. 45, 139-149 (2003).

[62] Stark, J A, "Adaptive image contrast enhancement using generalizations of histogram equalization," *Image Processing, IEEE Transactions on,* vol. 9, 889-896 (2000).

[63] Dale-Jones, R and Tjahjadi, T, "Four algorithms for enhancing images with large peaks in their histogram," *Image and Vision Computing,* vol. 10, 495-507 (1992).

[64] Hossain, F and Alsharif, M R, "Image Enhancement Based on Logarithmic Transform Coefficient and Adaptive Histogram Equalization," in *Convergence Information Technology, 2007. International Conference on*, 1439-1444 (2007).